Phased Array Antennas

Second Edition

R. C. HANSEN
Consulting Engineer R. C. Hansen, Inc.
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Phased Array Antennas

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This book is dedicated to those who made Microwave Scanning Antennas possible:

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Preface to the First Edition

Although array antennas have many decades of history, the last two decades have experienced a maturation, both in the understanding and design of arrays, and in the use of large sophisticated arrays. Radars utilizing electronic scanning arrays are in common use, from airport surveillance to missile detection and tracking; names of U.S. military systems, such as Aegis, Patriot, and Pave Paws, are well known. This book is a comprehensive treatment of all aspects of phased arrays; much has changed since the only other such work, *Microwave Scanning Antennas*, appeared in 1966. Most noteworthy has been the parallel development of inexpensive computer power and the theoretical understanding of nearly all aspects of phased array design. Design algorithms suitable for computers are emphasized here, with numerical tips and short algorithms sprinkled throughout the chapters. The work is prepared from the dual viewpoint of a design engineer and an antenna array analyst.

Chapter 2, on basic array characteristics, which covers grating lobes, quantization lobes, bandwidth, and directivity follows an introductory chapter. Highly efficient linear aperture and array synthesis techniques, including sum and difference patterns, are covered in Chapter 3. Chapter 4 treats synthesis of planar arrays. Array elements are covered in Chapter 5 and include not only the classic dipoles and slots, but TEM horns and patches. In Chapter 6, feeds for linear and planar arrays, both fixed beam and scanning, are examined; photonic time delay and feeders are included. Array performance is strongly affected by mutual impedance. Chapter 7 investigates ways of calculating this for various arrays elements, including an extensive treatment of ways of calculating array performance with mutual effects included. Among these are unit cell, spectral moment method, finite impedance matrix, and scattering techniques. Finite arrays are examined in Chapter 8, including the recently developed Gibbsian models. Next, Chapter 9 is an extensive view of superdirective arrays; the implications of high-temperature superconductors for antennas is an important feature. Multiple-beam arrays, as opposed to multiple-beam reflector feeds, are treated in Chapter 10.

Included are one- and two-dimensional Butler and Rotman lenses, and the practical meaning of beam orthogonality. Conformal arrays, ranging from ring arrays to arrays on cones, are covered next; much previously unpublished material is included in this chapter. Finally, Chapter 12 discusses array diagnostics, waveguide simulators in depth, and array tolerances. Extensive references to the archival literature are used in each chapter to offer additional sources of data.

ROBERT C. HANSEN

Tarzana, CA

Preface to the Second Edition

Several specialized types of phased arrays have attracted attention since the first edition. Connected dipole arrays offer wide bandwidth compared to a conventional array; these are discussed in detail in Chapter 12. (The old Chapter 12 is now Chapter 15). Reflectarrays provide reduced fabrication costs compared to a phased array. And retrodirective arrays offer interesting capabilities for data links. Both of these technologies are the subject of Chapter 13. The combination of reflectors and arrays is addressed in Chapter 14, both for focal plane arrays, including coma correction, and near-field Cassegrainian and Gregorian antennas.

Updates and additions have been made to existing sections: time delay deployment options for corporate fed arrays; fundamental limitations on Artificial Magnetic Conductors; Substrate Integrated Waveguide to replace rectangular waveguide; antennas for 60 GHz and beyond; impedances matching capabilities and limitations including Bode criterion limitations; elaboration of Scan Impedance and Scan Element Pattern calculations and measurements; and finally comments on completely overlapped sub-arrays.

CHAPTER ONE

Introduction

1.1 ARRAY BACKGROUND

Discovery of the first works on array antennas is a task best left to historians, but the two decades before 1940 contained much activity on array theory and experimentation. Some of the researchers were G. H. Brown, E. Bruce, P. S. Carter, C. W. Hansell, A. W. Ladner, N. E. Lindenblad, A. A. Pistolkors, S. A. Schelkunoff, G. C. Southworth, E. J. Sterba, and T. Walmsley. Primary journals were *Proc. IRE*, *Proc. IEE*, *BSTJ*, *RCA Review*, and *Marconi Review*. During World War II, much array work was performed in the United States and Britain. Interest in arrays returned in the early 1960s, with research projects at Lincoln Laboratories, General Electric, RCA, Hughes and others. Some of the array conferences are mentioned in the annotated reference list in Section 1.3.

A salient event was the publication by Academic Press of the three-volume book *Microwave Scanning Antennas* (*MSA*), with volume 1 appearing in 1964, and volumes 2 and 3 in 1966. This work was the first extensive coverage of phased arrays, with emphasis on mutual coupling theory, which is the basis of all array characteristics. After 30 years, *MSA* is still in print, through Peninsula Publishing.

It is the purpose of this book to present a thorough and extensive treatment of phased arrays, adding to and updating the array portions of *MSA*. The scope of the book is all types of arrays except adaptive, for which several excellent books exist; see references at the end of the chapter. Multiple-beam arrays are included. Because most arrays operate at frequencies that allow spacing above ground to be sufficiently large to preclude the ground affecting the array internal parameters, all arrays are presumed to be in free space. Active arrays, that is, those containing active devices, are not treated, nor are array-related circuit components, except for phasers, which are discussed briefly. It is also assumed that all array elements are identical, although the impedance matching may vary with the element position. A semantic difficulty

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arises with the phrase "phased array". For some people, this implies beam steering or scanning. But for others all arrays are phased; fixed beam broadside arrays are also phased. There are more important questions of terminology; these are addressed next.

1.2 SYSTEMS FACTORS

Important array factors for the systems designer are broadside pattern, gain versus angles, element input impedance, and efficiency. For all regular arrays, the pattern is given by the product of the element pattern and the pattern of the isotropic array, where the array elements are replaced by isotropes. However, the element excitations must be those of the real array; as discussed later, these are found by solving equations associated with a self-impedance and mutual-impedance or admittance matrix. In general, each element of an array will have a different input impedance. For a fixed beam array these are called "embedded impedances"; the obsolete and misleading term "active impedance" is deprecated. A scanning array not only has different element impedances, but each of them varies with scan angle. These element input impedances are called *scan impedances*.

The pattern of array gain versus angles is called *scan element pattern*; this term replaces *active element pattern*. The *scan element pattern* (SEP) is an extremely useful design factor. The element pattern and mutual coupling effects are subsumed into the *scan element pattern*; the overall radiated pattern is the product of the *scan element pattern* and the pattern of an isotropic array of elements scanned to the proper angle. The isotropic array factor incorporates the effects of array size and lattice, while the *scan element pattern*, as mentioned, incorporates element pattern, backscreen if used, and mutual coupling. Since the *scan element pattern* is an envelope of array gain versus scan angles, it tells the communications system or radar designer exactly how the array performs with scan, whether blind angles exist, and whether matching at a particular scan angle is advantageous. *Scan element pattern* is used for antenna gain in the conventional range equations. For an infinite array, the SEP is the same for all elements, but for a finite array each element sees a different environment, so that the SEP is an overall array factor. Use of infinite array *scan element patterns* allows array performance to be separated into this SEP and edge effects. Formulas for both finite array and infinite array *scan element pattern* are derived later; edge effects are also discussed later.

A similar parameter, appropriate for backscattering from antenna arrays, is the *scattering scan element pattern* (SSEP). This parameter gives the backscattered field intensity from an array element, when the array is excited by an incident plane wave. This then is different from the SEP, which relates radiated field intensity to total radiated power. The radar cross section (RCS) relates reradiated field intensity to incident field intensity, with a $4\pi R^2$ factor. The SSEP is this ratio of reradiated to incident intensity; a convenient normalization is to the broadside value. Just as in the case of a radiating array, the scattering array finite size and edge effects have been separated, so that the SSEP relates the effects of element design and array lattice. It can then be used to make design trades for type of element and lattice; the features

due to the array size are included simply by multiplying by the isotropic array factor. Of course, SSEP is related to the RCS pattern. It can be considered as the RCS pattern of one unit cell of the array.

System factors also arise in arrays used for wideband baseband (no carrier) applications. The one-way (communications) range equation, written without explicit wavelength dependence, is

$$P_r = \frac{P_t G A_e}{4\pi R^2} \tag{1.1}$$

where as usual P_r and P_t are received and transmitted powers, R is the range, and G and A_e are the gain of one antenna and the effective area of the other. Both gain and effective area include an impedance mismatch factor:[1] $(1-|\Gamma|^2)$. It is assumed that P_t is fixed, independent of frequency. If the GA_e product is relatively constant over the frequency band of interest, then the signal is transferred without significant dispersion, providing that the antenna and matching unit phase are well behaved also (Hansen and Libelo, 1995). Otherwise significant dispersion can occur.

From a casual look at array antennas, one might assume a planar array to be a constant effective area antenna. However, for a regularly spaced array of low-gain elements, as the frequency increases from nominal half-wave spacing, the gain increases until the first grating lobe appears, with the gain then dropping back to the original level. Further increases in frequency produce additional rises in gain followed by drops as grating lobes appear. The net result is that over a wide bandwidth the gain of an array is at best roughly constant and equal to the half-wave spaced value (Hansen, 1972). This does not include effects of embedded element impedance mismatch with frequency, a phenomenon that further greatly reduces gain. Thus the regularly spaced array is not a candidate for compensation of dispersion. An array with pseudorandom spacing does not experience the appearance of regular grating lobes as frequency is increased. The fraction of power in the sidelobes is roughly constant in a well-designed nonuniformly spaced array, and thus the gain is roughly constant with frequency. Of more importance, however, is the fact that very large numbers of elements are needed to achieve even moderately low sidelobe levels. Thus these types of arrays are not suitable for dispersion compensation either. Arrays of higher gain elements experience, in addition, the dispersion introduced by the elements themselves and are even less suitable.

1.3 ANNOTATED REFERENCE SOURCES

Many textbooks discuss arrays, but the books and digests listed here provide in-depth resources on phased arrays.

[1]Note that "effective length", which is defined as open circuit voltage divided by incident electric field, does not include impedance mismatch, and is therefore useless by itself.

Microwave Scanning Antennas, R. C. Hansen, Ed., 3 vols., Academic Press, 1964, 1966 [Peninsula Publishing, 1985, 442 pp., 400 pp., 422 pp. (Peninsula combined volumes)].

This, the first extensive work on phased arrays, is still quite useful. Volume 1 has a chapter on aperture distributions. Volume 2 includes array theory, and infinite and finite array analysis; probably the first development of the spectral domain analysis technique for arrays. Feeds, frequency scanning, and multiple beams are covered in vol. 3; multiple beams by Butler of matrix fame.

Proceedings of the 1964 RADC Symposium on Electronically Scanned Array Techniques and Applications, report RADC-TDR-64-225, AD-448 481.

Contained here are early papers on phase quantization errors, ferrite and semiconductor phasers, and beam forming matrices.

The Theory and Design of Circular Antenna Arrays, James D. Tillman, University of Tennessee Engineering Experiment Station, 1966, 235 pp.

This treatise on ring arrays and concentric ring arrays applies sequence theory of azimuthal modes, called symmetrical components in electric power work, to the analysis of impedance and pattern. Array scanning is also discussed.

Proceedings of the 1970 NELC Conformal Array Conference, TD-95, Naval Electronics Lab. Center, AD-875 378.

Both ring arrays and cylindrical arrays are treated in papers, both theoretically and for applications.

Phased Array Antennas, A. A. Oliner and G. H. Knittel, Artech, 1972, 381 pp.

This book is a record of the 1970 Phased Array Antenna Symposium held at Polytechnic Institute of Brooklyn. Included are many papers on impedance calculations, blind angles, and so on, and also on practical aspects, such as scan compensation and feeding and phasing.

Theory and Analysis of Phased Array Antennas, N. Amitay, V. Galindo, and C. P. Wu, Wiley–Interscience, 1972, 443 pp.

Arrays of waveguide radiators are the subject here. The spectral domain method is used extensively. Small finite arrays are solved via equations over the modes and elements. This work is one of the first using multimode spectral analysis.

Proceedings of the 1972 NELC Array Antenna Conference, TD-155, 2 Parts, Naval Electronics Lab. Center, AD-744 629, AD-744 630.

Many papers cover array techniques and components; adaptive arrays, and conformal arrays.

Theory and Application of Antenna Arrays, M. T. Ma, Wiley–Interscience, 1974, 413 pp.

Transform analysis and synthesis of fixed beam arrays is covered, along with many general array examples. The effect of ground on arrays represents a significant part of this book.

Conformal Antenna Array Design Handbook, R. C. Hansen, Ed., Naval Air Systems Command, 1982, AD-A110 091.

This report summarizes a decade of Navair-supported work on cylindrical and conical slot arrays, including mutual impedance algorithms.

Antenna Theory and Design, R. S. Elliott, Prentice-Hall, 1981, 594 pp.

This text is an excellent source for waveguide slot array analysis and synthesis. Sidelobe envelope shaping is treated in detail.

The Handbook of Antenna Design, A. W. Rudge, K. Milne, A. D. Olver, and P. Knight, Eds., IEE/Peregrinus, 1983, vol. 2, 945 pp.

This handbook contains chapters on linear arrays, planar arrays, conformal arrays, ring arrays, and array signal processing. Extensive data are included on array analysis and synthesis, including mutual coupling effects.

Proceedings of the 1985 RADC Phased Array Symposium, H. Steyskal, Ed., report RADC-TR-85-171, AD-A169 316.

This symposium record contains papers on microstrip arrays, adaptive arrays, and scan impedance, among others. A second volume has restricted distribution.

Antenna Handbook, Y. T. Lo and S. W. Lee, Van Nostrand Reinhold, 1988.

This handbook contains chapters on array theory, slot arrays, periodic and aperiodic arrays, practical aspects, and multiple-beam arrays.

Antenna Engineering Handbook, R. C. Johnson and H. Jasik, McGraw-Hill, 1993.

This updated edition of an old classic contains chapters on array theory, slot arrays, frequency scan and phased arrays, and conformal arrays.

Phased Array Antenna Handbook, R. J. Mailloux, Artech, 1994, 534 pp.

This specialized handbook covers most array topics, with emphasis on analysis and synthesis. A chapter covers limited scan arrays and time delayed arrays.

Phased Array-Based Systems and Applications, N. Fourikis, Wiley–Interscience, 1997.

This book emphasizes systems aspects of arrays.

1.3.1 Adaptive Antenna Reference Books

Compton, R. T., Jr., *Adaptive Antennas*, Prentice-Hall, 1988.

Hudson, J. E., *Adaptive Array Principles*, IEE/Peregrinus, 1981.

Monzingo, R. A. and Miller, T. W., *Introduction to Adaptive Arrays*, Wiley, 1980.

Widrow, B. and Stearns, S. D., *Adaptive Signal Processing*, Prentice-Hall, 1985.

REFERENCES

Hansen, R. C., "Comparison of Square Array Directivity Formulas", *Trans. IEEE*, Vol. AP-20, Jan. 1972, pp. 100–102.

Hansen, R. C. and Libelo, L. F., "Wideband Dispersion in Baseband Systems", *Trans. IEEE*, Vol. AES-31, July 1995, pp. 881–890.

CHAPTER TWO

Basic Array Characteristics

This chapter is concerned with basic characteristics of linear and planar arrays, primarily with uniform excitation. The theory of, and procedures for, the design of array distributions to produce narrow-beam, low-sidelobe patterns, or shaped beams, are covered in detail in Chapter 3. Impedance effects due to mutual coupling are treated in Chapter 7. Covered here are such parameters as pattern, beamwidth, bandwidth, sidelobes, grating lobes, quantization lobes, and directivity.

2.1 UNIFORMLY EXCITED LINEAR ARRAYS

2.1.1 Patterns

In general, the excitation of an array consists of an amplitude and a phase at each element. This discrete distribution is often called an aperture distribution, where the discrete array is the aperture. The far-field radiation pattern is just the discreet Fourier transform of the array excitation. The array pattern is the product of the isolated element pattern and the isotropic array factor; this is the "forced excitation" problem. To achieve this, the element drives are individually adjusted so that the excitation of each element is exactly as desired. More common is the "free excitation" situation, where the element drives are all fixed, and the element excitations are those allowed by the *scan impedance*. The latter is discussed in detail in Chapter 7. Here the concern will be only with the forced excitation array, where the excitations are constant in amplitude, but may have a scan phase.

A common notation in the antenna literature is used here, where λ is wavelength, d is element spacing, $k = 2\pi/\lambda$, and the angular variable is u. The latter is $u = (\sin\theta - \sin\theta_0)$ where θ_0 is the scan angle. Uniform (equal spacing) is assumed in this chapter; unequally spaced arrays are discussed in Chapter 3. Although it is simpler

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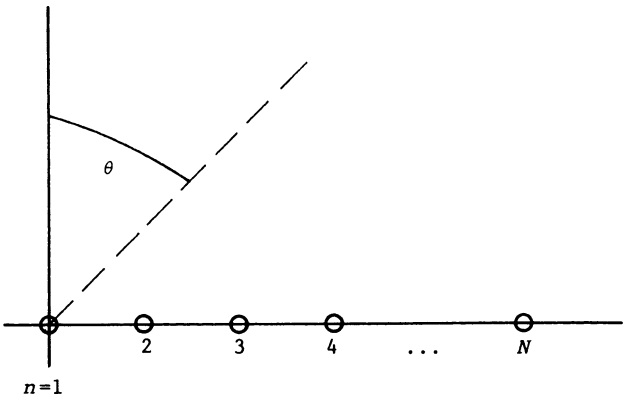

FIGURE 2.1 Linear array geometry.

to have a coordinate system axis in the center of a linear array, complications ensue for even and odd numbers of elements. A more general case starts the coordinate system at one end of the array, as shown in Figure 2.1. The pattern, sometimes called a space factor, is

$$F(u) = \sum A_n \exp[jkd(n-1)u] \tag{2.1}$$

A_n is the complex excitation, which for much of this section will be assumed constant.

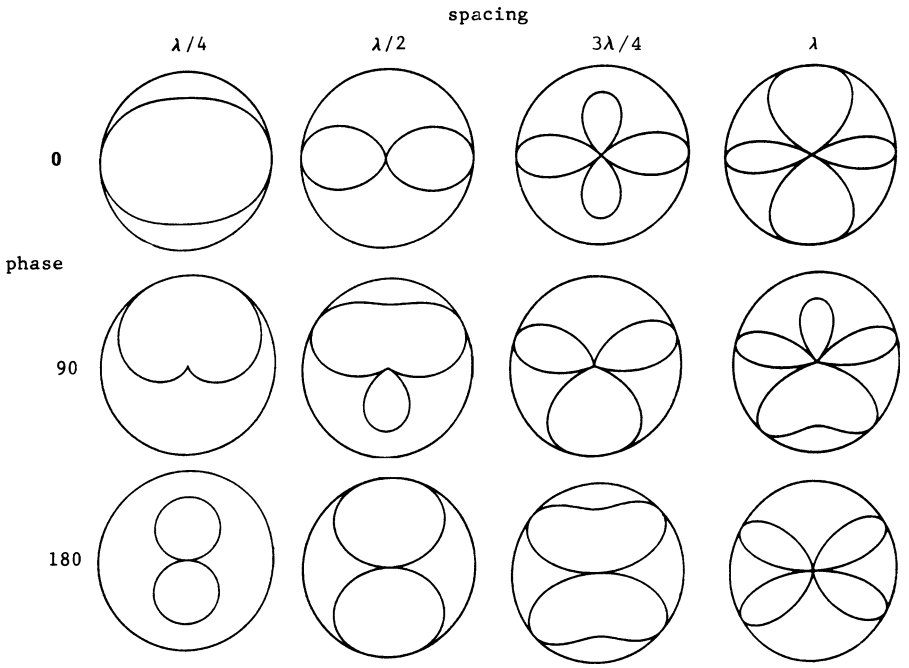

FIGURE 2.2 Two-element array patterns.

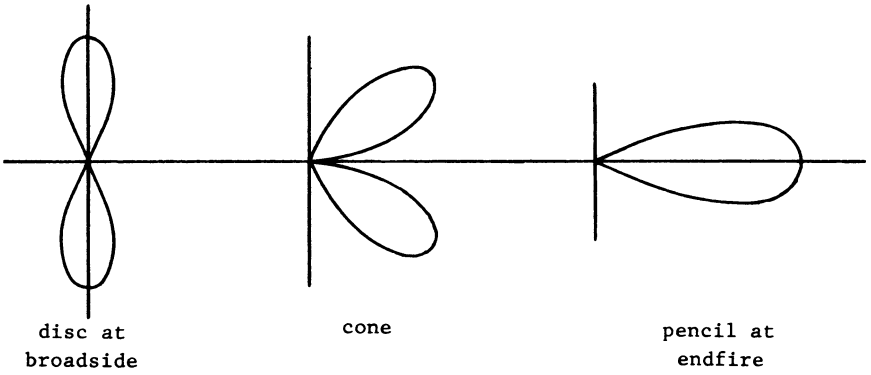

FIGURE 2.3 Linear array beams.

For uniform excitation, the array pattern becomes a simple result, where the exponential in Eq. (2.2) can be discarded, leaving a real pattern times an exponential:

$$F(u) = \exp[j\pi(N-1)u]\frac{\sin\frac{1}{2}Nk\,du}{N\sin\frac{1}{2}k\,du} \tag{2.2}$$

This interelement phase shift is $kd\sin\theta_0$. By varying this phase shift, the beam position can be scanned. Figure 2.2 shows patterns produced by the various spacings and phases (Southworth, 1930).

Many linear arrays are designed to produce a narrow beam. Figure 2.3 depicts how the beam changes with scan. With no scan the narrow beam is omnidirectional around the array axis. As the beam is scanned, this "disk" beam forms into a conical beam as shown in the center sketch. When the 3-dB point gets to 90°, a singular situation occurs. Beyond this scan angle the beam has two peaks, and the "beamwidth" will double as the outside 3-dB points are used. Finally, at endfire a pencil beam results; thus a linear array at broadside yields directivity in one dimension while at endfire it yields directivity in two dimensions. It might be expected as a result that the endfire beamwidth is broader; this will be shown next.

2.1.2 Beamwidth

The half-power points on a uniform array pattern are found by putting $\sin\frac{1}{2}Nk\,du_3/(N\sin\frac{1}{2}k\,du_3) = \sqrt{0.5}$. Figure 2.4 gives the solution of this as a function of the number of elements in the array. For $N \geq 7$, the variation in normalized beamwidth Nu_3 is $<1\%$, and the error is only 5% for $N = 3$. Thus for large arrays, the half-power points are given simply by $\frac{1}{2}Nk\,du_3 = \pm 0.4429$. For a beam scanned at angle θ_0, this gives the 3-dB-beamwidth θ_3 as

$$\theta_3 = \arcsin\left(\sin\theta_0 + 0.4429\frac{\lambda}{Nd}\right) - \arcsin\left(\sin\theta_0 - 0.4429\frac{\lambda}{Nd}\right) \tag{2.3}$$

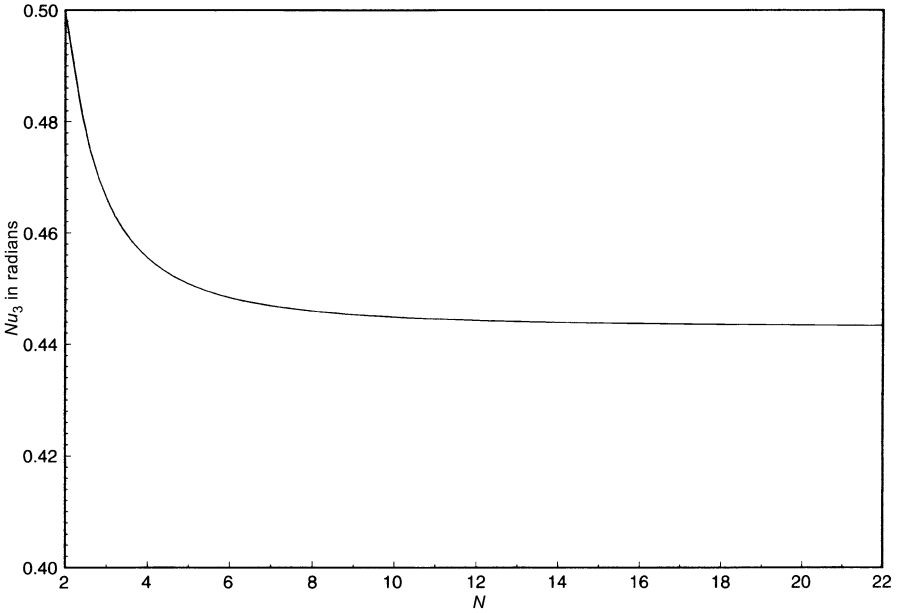

FIGURE 2.4 Normalized beamwidth versus number of elements.

For large N, this reduces to

$$\theta_3 \simeq \frac{0.8858\lambda}{Nd\cos\theta_0} \tag{2.4}$$

The beam collapse near endfire, where the 3-dB point is at 90°, occurs for a scan angle of

$$\theta_0 = \arcsin\left(1 - 0.4429\frac{\lambda}{Nd}\right) \tag{2.5}$$

The beamwidth broadening near endfire is shown in Figure 2.5 for several arrays. For large N, the endfire beamwidth is

$$\theta_3 \simeq 2\sqrt{\frac{0.8858\lambda}{Nd}} \tag{2.6}$$

The accuracy of this is better than 1% for $Nd/\lambda > 4$. The endfire beamwidth is larger than the broadside value by $2.14\sqrt{Nd/\lambda}$. Thus the endfire pencil beam is broader than the broadside pancake beam.